Exam Quantum Field Theory
February 10, 2015
Start: 9:00h End: 12:00h

Each sheet with your name and student ID

INSTRUCTIONS: This is a closed-book and closed-notes exam. You are allowed to bring one A4 page written by you on one side, with useful formulas. The exam duration is 3 hours. There is a total of 9 points that you can collect.

NOTE: If you are not asked to **Show your work**, then an answer is sufficient. However, you might always earn more points by answering more extensively (but you can also loose points by adding wrong explanations). If you are asked to **Show your work**, then you should explain your reasoning and the mathematical steps of your derivation in full. Use the official exam paper for *all* your work and ask for more if you need.

USEFUL FORMULAS

The energy projectors for spin 1/2 Dirac fermions:

$$\sum_{r=1,2} u_r(\vec{p})\bar{u}_r(\vec{p}) = \frac{\not{p}+m}{2m}$$

$$\sum_{r=1,2} v_r(\vec{p})\bar{v}_r(\vec{p}) = \frac{\not{p}-m}{2m}$$

$$\{\gamma_5, \gamma_\mu\} = 0 \qquad \gamma_5^2 = \mathbb{1}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu} \quad (d=4)$$

1. (2 points) The path integral for the $\lambda\phi^4$ theory coupled to an external scalar source J can be written as follows

$$Z[J] = Z[J=0,\lambda=0]\, e^{-\frac{i\lambda}{4!}\int d^4w\left(\frac{\delta}{\delta iJ(w)}\right)^4} e^{-\frac{i}{2}\int\int d^4x d^4y\, J(x)D(x-y)J(y)}$$

when expanded perturbatively in the coupling λ. Derive the analogous expression for the Yukawa theory with Lagrangian density

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 - g\,\bar{\psi}\psi\phi \qquad (1)$$

with $\partial\!\!\!/ = \gamma^\mu\partial_\mu$, m the fermion mass, M the scalar mass, and g the Yukawa coupling. **Show your work**

2. (2 points total) Consider the Yukawa theory (1) in d spacetime dimensions,

a) (1 points) Show that the canonical dimension of the Yukawa coupling g is given by

$$[g] = \frac{4-d}{2}$$

b) (1 points) For which spacetime dimension d is the theory renormalizable? Show your arguments and make use of the fact that the superficial degree of divergence for this theory is

$$D = d - \left(\frac{4-d}{2}\right)V - \left(\frac{d-2}{2}\right)E_B - \left(\frac{d-1}{2}\right)E_F$$

with V the number of vertices, E_B the number of external bosonic lines and E_F the number of external fermion lines.

3. (2 points total) Under a chiral transformation a spin-1/2 Dirac fermion field transforms as

$$\psi'(x) = e^{i\alpha\gamma_5}\,\psi(x)$$

with α a real parameter of the transformation.

a) (1 points) Find the transformation for the field $\bar{\psi}(x)$. **Show your work**

b) (1 points) Determine which of the two terms in the Lagrangian density for a free spin-1/2 Dirac fermion

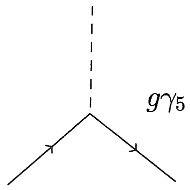
$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi$$

is not invariant under the chiral transformation. **Show your work**

4. (3 points total) The Yukawa theory with a γ^5-type interaction has the following Lagrangian density

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 - ig\bar{\psi}\gamma^5\psi\phi\,,$$

(the factor i in the interaction guarantees hermiticity) with m the fermion mass, M the scalar mass, and g the Yukawa coupling. The Feynman rule for the vertex is



a) (2 points) Calculate the unpolarized squared amplitude $X = (\mathcal{A}^\dagger\mathcal{A})_{\text{unpol.}}$ for the decay process $\phi(p) \to f^-(p_1)f^+(p_2)$ at leading order in g. **Show your work**

b) (1 points) Using relativistic kinematics find the decay rate Γ in the centre-of-mass (CM) frame where the decaying particle is at rest, i.e. $s = p^2 = M^2$, and show that it can be written in terms of m and M only. What happens for $M < 2m$? **Show your work**

   Use the CM formula for the decay rate

$$\Gamma = \frac{1}{4\pi}\frac{m^2}{M}\sqrt{1 - \frac{4m^2}{M^2}}\,X$$

   with X defined in a).